SQUARES AND HEXAGONS OF FORMAL-ETHICAL OPPOSITION IN TWO-VALUED ALGEBRA OF FORMAL ETHICS. AN ALGEBRAIC SYSTEM OF MORAL-EVALUATION-FUNCTIONS: "BEING", "KNOWING", "FAITH", "LOVE", ET AL.

Prof. Hab. Dr. Vladimir Lobovikov[1]
Ural Federal University, Yekaterinburg, Russia

Resumen: El artículo presenta resultados de una investigación ético-formal sobre conocimiento, fe, duda, amor, odio, indiferencia, temor y algunas otras categorías morales, como funciones de evaluación moral determinadas por una o dos variables de evaluación moral, tomando sus valores desde el conjunto {g (good), b (bad)} en un álgebra de una ética formal o de rigor moral de dos valores. Dado que las evaluaciones morales son relativas a un evaluador moral, el artículo utiliza el álgebra mencionada para elaborar una solución al viejo problema de la relatividad de la evaluación moral. Usando la solución ofrecida para el problema de la relatividad dentro del marco del álgebra de la ética formal, el autor da definiciones tabulares precisas de la funciones de evaluación moral en cuestión. Las relaciones ético-formales entre las funciones son representadas analíticamente por sistemas de ecuaciones ético-formales de esa álgebra y también gráficamente por medio de cuadrados y hexágonos de oposición ético-formal. Estos modelos algebraicos y gráficos de relaciones ético-formales entre categorías morales bajo consideración se perfeccionan unos con otros en relaciones heurísticas y pedagógicas. Por lo tanto, ambas son valiosas de desarrollar posteriormente.

Descriptores: Bueno · Malo · Relatividad de la evaluación moral · Álgebra de la ética formal · Función · Variable · Ética formal · Cuadrado de la oposición ·Fe· Duda · Amor · Odio · Indiferencia · Temor.

Abstract: The paper presents results of a formal-ethical investigation of knowledge, faith, doubt, love, hate, indifference, fear, and some other moral categories as moral-evaluation-functions determined by one or two moral-evaluation-variables taking their values from the set {g (good), b (bad)} in two-valued algebra of formal ethics of moral rigor. As moral-evaluations are relative to some moral-evaluator, the paper utilizes the mentioned algebra for elaborating a solution of the old problem of moral-evaluation-relativity. Using the submitted solution of the relativity problem within the framework of algebra of formal ethics the author gives precise tabular definitions of the moral-evaluation-functions in question. The formal-ethical relations among the functions are represented analytically by systems of formal-ethical equations of that algebra and also graphically by squares and hexagons of formal-ethical opposition. These algebraic and graphic models of formal-ethical relationship among the moral categories under consideration enhance each other in both heuristic and pedagogic relations. Therefore both are worth developing further.

Keywords: Good· Bad · Moral-evaluation-relativity ·Aalgebra-of-formal-ethics ·Ffunction · Variable ·Formal-ethical · Square-of-opposition ·Knowledge · Faith ·Doubt · Love · Hate, · Indifference · Fear.

Enviado: 02/09/2014. Aceptado: 20/11/2014

[1] Department of Ontology and Theory of Cognition. TheUral Division of the Russian Academy of Sciences. E-mail: vlobovikov@mail.ru

1. Two-valued algebra of formal ethics of moral rigor as a fundamental generalization of the classical algebra of formal logic

In the history of human culture, initially the square of opposition was invented and used as a tool for formal logic analysis of thinking and as a handy pedagogical means (visual model) for teaching logic (Correia, M. 2009, 2012, 2014). Therefore very often the square was called the logical one: as a rule, it was an object of professional interest of logicians and mathematicians. They discovered (or invented) many non-standard but theoretically interesting and practically useful interpretations of the logical square, investigated its reducing to a triangle and developed it to cubes, pentagons, hexagons, octagons, *n-gons*, etc. (Béziau, J.-Y. 2012a,b; Béziau, J.-Y. and Payette, G. 2012; Desclés, J.-P. and Pascu, A. 2012; Dufatanye, A.-A. 2012). This development existed within the framework of subject-matter of formal logic dealing with thought and speech. However in 21th century it was recognized that the square of opposition is significant not only for logic as such, but also for many other normative disciplines dealing with structural-functional investigating any culture of human activity (ethics, aesthetics, etc.). In this respect the reinterpreted opposition squares and hexagons have a fundamental significance for constructing AI-systems (See Prade, H. 2014). For adequate constructing and transforming the internal world-views of AI-robots, it is necessary to have a substantially generalized conception of the n-gons of opposition. For instance, to realize adequate moral-legal programming AI-robots acting among humans it is necessary to go from the formal logic to formal ethics (Lobovikov, V. 2009a,d,e, 2012, 2013), in particular, from logical squares and hexagons of thought and speech to ethical squares and hexagons of human activity in general (Lobovikov, V. 2010a,b, 2011b, 2014b). Also for adequate moral-legal programming AI-robots it is worth going from the formal logic of legal norms (deontic logic) to a discrete mathematical model of the natural law system of juridical modalities – algebra of the natural law (Lobovikov, V. 2007a,2010b,c, 2011a,b).

In the fundamental survey of history of the famous Lvov-Warsaw School of Philosophy, the well-known Polish logician Woleński, J. (2014) writes: "According to Łukasiewicz, "Logic is morality of thought and speech"". In the present paper I intend to develop further the profound idea expressed in the mentioned statement of Jan Łukasiewicz. I accept this idea and try to investigate its consequences. First of all it is relevant to fix the fact that thought-and-speech is a *specific particular case* of human activity in general regulated by morals. Hence, logic is a particular case of morality in general. Ethics studies moral regulation of any human activity. Consequently, ethics is more universal and fundamental than logic. Formal logic is nothing but formal ethics of thought-and-speech. True and false propositions are

specific particular cases of good and bad actions, respectively. Thus truth-values of propositions are specific particular cases of moral values of actions. Formal-logical contradictions are particular cases of formal-ethical ones. Formal-logical entailment among thoughts is a specific kind of formal-ethicalentailment among moral actions and moral-action-forms deprived of their specific contents. From this point of view, systematically developing the idea of Łukasiewicz, it is quite natural to arrive to the conclusion that, using the precedent made by contemporary symbolic logic leads to creating a mathematized symbolic ethics. If this is accepted then, naturally, the well-known two-valued algebra of classical logic is a modest particular case of hypothetical two-valued algebra of formal ethics of moral rigor.

The present paper is aimed at explicating universal and immutable laws of the world of pure moral *values* represented as an algebraic system. However many humans believe that universal and immutable laws of the world of pure *values* do not exist. The overwhelming majority believes that being of such *formal-axiological laws* is impossible as *values and assessments are necessarily relative* and undergo permanent change. Many humans believe that any relativism is incompatible with objective knowledge. In particular, they think that if moral evaluations of concrete contents of moral actions (and agents) are *relative to evaluators*, then objective moral laws (=necessarily universal and immutable positive-moral-evaluations of actions) do not exist as they are impossible on principle. However, in my opinion, objective knowledge is compatible with some forms of relativism. For instance, it is compatible with a *relative relativism*, i.e. such relativism, which is not an absolute one.

This general statement may be exemplified by the *relativistic* physics. In the special-relativity-theory it is demonstrated that (if *x* is a physical body then) mass of *x*, length of *x*, time of *x* are *necessarily relative*: they necessarily depend upon that physical systems, in relation to which they are measured; measuring in relation to different systems gives different results. Nevertheless there are some physical qualities, which invariantly exist in relation to all physical systems. These *invariant* physical qualities are considered as objective laws of the special-relativity-theory. Thus physics has made a *precedent* to be applied to *analogous* cases. The situation in relativistic ethics is *analogous* to the one in relativistic physics. Therefore the experience of creating relativistic physics is heuristically important for creating relativistic ethics as a system of objective knowledge of absolute laws of the moral-value-relativity.

The special-relativity-theory was precisely formulated and developed by means of the mathematical language. It was impossible to create and develop this theory by means of the natural language. In formal ethics the situation is *analogous* to the one in physics. It is impossible to create and develop a theory of relativity of

moral evaluations (as a system of absolute laws of their relativity) at the level of natural language. For departing from the old-fashioned absolute-moral-relativism to precise mathematical formulations of absolute formal-ethical laws of moral-evaluation-relativity, it is necessary to construct an artificial language of formal ethics for investigating a mathematical model of the system of moral evaluations of actions and agents. Hereafter let us start constructing the artificial language and the mathematical model.

Below the possibility of mathematical representation of moral activity is demonstrated by the elementary mathematical ethics of moral rigor — two-valued algebra of good and evil. This algebra is based upon the set of moral acts and agents. By definition, moral acts are such and only such operations, which are either good, or bad ones in the moral meaning of the words "good" and "bad". Algebraic operations defined on the set of moral acts and agents are moral-evaluation-functions. Moral-evaluation-variables of these functions take their values from the set *{g, b}*. Here the symbols "*g"* and "*b"* stand for the moral values "good" and "bad", respectively. The functions take their values from the same set. The symbols: "*x"* and *"y"* stand for moral-forms of acts. Elementary moral-act-forms deprived of their contents are independent moral-evaluation-variables. Compound moral-act-forms deprived of their contents are moral-evaluation-functions determined by these variables.

Let symbol Σ stand for the *moral evaluator*, i.e. that person (individual or collective one – it does not matter), in relation to which all evaluations are generated. In the moral-evaluation-relativity theory, Σ is a variable: changing values of the variable Σ can result in changing moral evaluations of concrete acts and agents. However if a value of the variable Σ is fixed, then moral evaluations of concrete acts and agents are definite.

DEFINITION DF-1: in two-valued algebra of formal ethics of moral rigor, for any moral actions and agents x, y, Σ, the below-given table 1 is a precise definition of the moral-functional sense of the binary moral operations introduced by the glossary 1.

The glossary for the below-given table 1: The symbol K_2xy stands for the moral operation "combining (uniting) *x* and *y* (in the conduct as a whole)". S_2xy stands for moral operation "separating *x* and *y"*. A_2xy stands for "non-excluding choice of the best among the acts made up by *x* and *y*". W_2xy — "abstaining from both *x* and *y*". U_2xy — "excluding choice of the best between the acts *x* and *y*". T_2xy means "moral identifying *x* and *y*". C_2xy means the moral operation "doing *y* in response to *x*". H_2xy means "offensive, attack, assault of y on (against) *x*". D_2xy stands for "*x's* defense of (what, whom) *y*". In other words, D_2xy means "*x's* conservation of *y*".

Table 1: Binary moral operations of algebra of formal ethics

		1	*2*	*3*	*4*	*5*	*6*	*9*	*7*	*8*
x	*y*	K_2xy	S_2xy	A_2xy	W_2xy	U_2xy	T_2xy	C_2xy	H_2xy	D_2xy
g	g	g	b	g	b	b	g	g	b	g
g	b	b	g	g	b	g	b	b	b	g
b	g	b	g	g	b	g	b	g	g	b
b	b	b	g	b	g	b	g	g	b	g

DEFINITION DF-2: in two-valued algebra of formal ethics of moral rigor, for any moral actions and agents y, Σ, the below-given table 2 is a precise definition of the moral-functional sense of the unary moral operations introduced by the glossary 2.

The glossary for the below-given table 2: Let symbol *By* stand for the moral-evaluation-function "existence of y". *Ny* – "nonexistence of y". *Ey* – "episteme (alethic knowledge) of (what, whom) y". *Fy* – "faith (not-revisable one) in (what, whom) y". *Sy* – "tolerating (what, whom) y". *Hy* – "non-tolerating (what, whom) y". *Ty* – "(moral) tolerance to y, i.e. standing both: y and not-y". *Iy* – "(moral) indifference to (what, whom) y". *Dy* – "(moral) non-indifference to y". *Gy* – "(moral) non-tolerance to y".

Table 2: Unary moral operations

y	*By*	*Ny*	*Ey*	*Fy*	*Sy*	*Hy*	*Ty*	*Iy*	*Dy*	*Gy*
g	g	b	g	g	g	b	b	b	g	g
b	b	g	b	b	b	g	b	b	g	g

DEFINITION DF-3 (of *formal-ethical-equivalence*-relation):in two-valued algebra of formal ethics, moral-evaluation-functions Ω and Δare *formally-ethically equivalent* (this is represented by the symbol "Ω=+=Δ"*),* if and only if they acquire identical moral values (from the set {*g, b*}) under any possible combination of moral values of the variables.

DEFINITION DF-4 (of *formal-ethical contradiction*):in two-valued algebra of formal ethics, a moral-evaluation-function Ω is called *formally-ethically (or invariantly) bad* one, if and only if it acquires the moral value *b (bad)* under any possible combination of moral values of its variables. In other words, *moral activity form* Ω is *formally-ethically inconsistent one*, iff Ω=+=b.

DEFINITION DF-5 (of *invariant law* of moral-relativity theory): in two-valued algebra of formal ethics, a moral-evaluation-function is called *formally-ethically*

(or invariantly) good one (or a *law of formal ethics*), if and only if the function acquires the moral value *g (good)* under any possible combination of moral values { *g (good), b (bad)* } of its evaluation-variables. In other words, *moral activity form*Ω isa (universal and immutable) *law of formal ethics*, iff Ω=+=g.

DEFINITION DF-6: (of *formal-ethical-consequence*-relation):in two-valued algebra of formal ethics, a moral-evaluation-function Δ is called a *formal-ethical consequence* of a moral-evaluation-function Ω, if and only if C_2ΩΔ=+=g. In other words, by definition, "moral-action-form Ω *formally-ethically entails (or formally-ethically implies)* moral-action-form Δ", if and only if C_2ΩΔ=+=g. In this case one may say that "Δ *formally-ethically follows from* Ω".

Taking into an account the above-given definitions, one can make an important discovery: the *formal-ethical laws* of moral-relativity theory do not depend upon possible changes of the moral evaluator Σ. If and only if Ω is a formal-ethical law, then Ω is morally good in relation to every moral evaluator Σ. Moreover, in the moral-relativity theory under review, *formal-ethical contradictions* of complex moral conduct also do not depend upon possible changes of the moral evaluator Σ. If and only if Ω is a formal-ethical contradiction, then Ω is morally bad in relation to every moral evaluator Σ. Finally, if there is the above-defined *formal-ethical equivalence*-relation between moral-evaluation-functions Ω and Δ, then the functions Ω and Δ are *formally-ethically equivalent* ones in relation to every moral evaluator Σ. Hence, in spite of the flexibility and relativity of moral evaluations, there are *absolute invariants* (immutable universal laws) of the moral relativity. Thus the moral relativity is not absolute but relative one.

2. An algebraic system of moral-evaluation-functions: "Being", "Knowing", "Faith", "Love", "Fear", organized by squares and hexagons of formal-ethical opposition

The glossary for the below table 3: The symbol K^Exy stands for the moral-evaluation-function "*x*'s *knowing* (what, whom) *y* (in the proper *episteme* meaning of the word)", or "making *y* an *absolute knowledge* (*episteme)* of (for) *x*". The symbol A^Dxy stands for the moral-evaluation-function "*x*'s *admitting (assuming) y (as an episteme)*", or "making *y admissible (assumable) for x.* (In other words, A^Dxy *means y's being assumed as an episteme by x.*) I^Dxy stands for "making *y inadmissible (not assumable) as an episteme for x". N^Exy – "*x*'s *not-knowing* (what, whom) *y*, or "*y's being not an absolute-knowledge (=episteme), but an ignorance* of (for) *x*". D^Xxy – "*y's* being *an opinion* (*doxa)* of (for) *x"*, i.e. "neither *x*'s *knowingy* (*as an episteme)*, nor *x*'s *knowingnot-y (as an episteme)*". D^Gxy – "*x's dogmatism concerning y"*, i.e. "either *x*'s *knowingy* (*as an episteme)*, or *x*'s *knowingnot-y (as an episteme)*". The

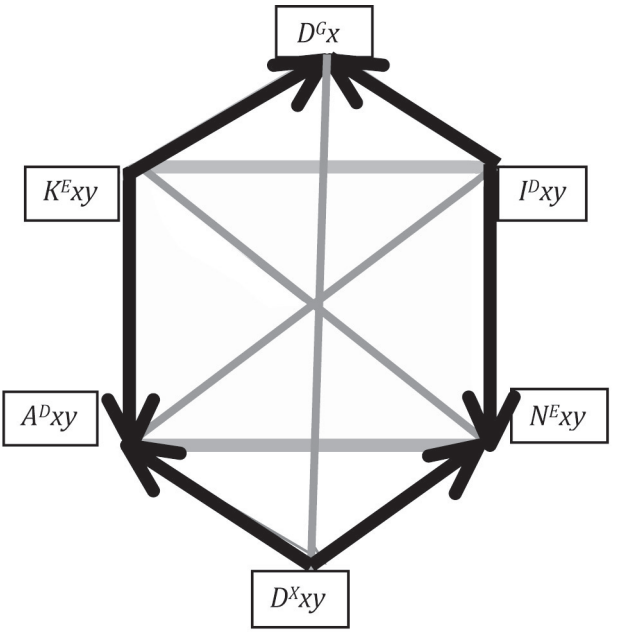
moral-evaluation-functional sense of these operations of algebra of formal ethics is defined by the following table 3. Previously this tabular definition was published in (Lobovikov, V. 2009b, 2011c,f,g).

Table 3: Binary operations "episteme" and "assumption"

x	y	K^Exy	A^Dxy	I^Dxy	N^Exy	D^Xxy	D^Gxy
g	g	b	g	b	g	g	b
g	b	b	g	b	g	g	b
b	g	g	g	b	b	b	g
b	b	b	b	g	g	b	g

The formal-ethical relationship among these operations is represented below by the following formal-ethical hexagon containing the square of formal-ethical opposition as its aspect.

Picture 1: The Formal-Ethical Hexagon and the Square of Formal-Ethical Opposition 1



The formal-ethical relations between moral *subalterns*, represented in this picture by *arrows*, also could be represented analytically by the following equations of algebra of formal ethics.

1) $C_2K^{E}xyA^{D}xy{=}{+}{=}g.$

2) $C_2I^{D}xyN^{E}xy{=}{+}{=}g.$

3) $C_2D^{X}xyN^{E}xy{=}{+}{=}g.$

4) $C_2D^{X}xyA^{D}xy{=}{+}{=}g.$

5) $C_2K^{E}xyD^{G}xy{=}{+}{=}g.$

6) $C_2I^{D}xyD^{G}xy{=}{+}{=}g.$

The formal-ethical *contradictoriness*relations between moral *opposites*, represented in the picture 1 by *lines crossing the square*, also could be represented analytically by the following equations of algebra of formal ethics.

7) $U_2K^{E}xyN^{E}xy{=}{+}{=}g.$ 8) $U_2I^{D}xyA^{D}xy{=}{+}{=}g.$ 9) $U_2D^{X}xyD^{G}xy{=}{+}{=}g.$

As to the formal-ethical *contrariness* relation (between$K^{E}xy$and $I^{D}xy$), represented in the picture 1 by the *upper horizontal line of the square*, the following equation of algebra of formal ethics is true.

10) $K_2K^{E}xyI^{D}xy{=}{+}{=}b.$ (But the equation $A_2K^{E}xyI^{D}xy{=}{+}{=}g$ is false.)

Concerning the formal-ethical *sub-contrariness* relation (between $A^{D}xy$ and $N^{E}xy$), represented in the picture 1 by the *lower horizontall line of the square*, the following equation of algebra of formal ethics is true.

11) $A_2A^{D}xyN^{E}xy{=}{+}{=}g.$ (But the equation$K_2A^{D}xyN^{E}xy{=}{+}{=}b$ is false.)

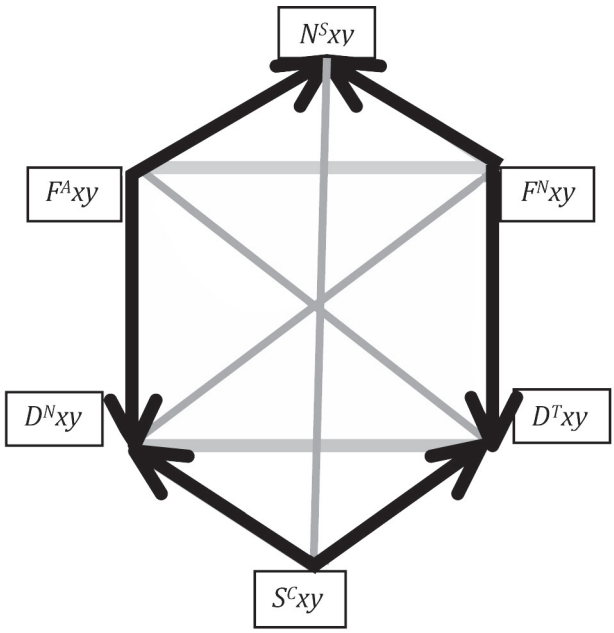
The glossary for the below table 4: Let symbol $F^{I}xy$ stand for moral-evaluation-function "x's *alethic* (true) *faith (not-revisable belief)* in (what, whom) *y*". The symbol $D^{N}xy$ stands for moral-evaluation-function "x's *alethic (true) doubt (not-removable one)* in *not-y*", i.e. "nonbeing of "x's *true faith (not-revisable belief)* in *not-y*". $F^{N}xy$ stands for the function "x's *true faith (not-revisable belief)* in *not-y*". $D^{T}xy$ – "x's *alethic (true) doubt (not-removable one)* in *y*". In other words, $D^{T}xy$ stands for "nonbeing of "x's *alethic faith (not-revisable belief)* in *y*". $S^{C}xy$ – "x's *alethic (true) skepticism concerning y, i.e.* x's *doubt (not-removable one) in both: y and not-y". $N^{S}xy$ –* "nonbeing of x's *alethic skepticism concerning y"*, i.e. *"*x's *fanaticism concerning y"*, or, in other words, "*either* x's *alethicfaith (not-revisable belief)* in *y*", or "x's *alethic faith* in *not-y*". The moral-evaluation-functional sense of the operations is defined below by the table 4(See Lobovikov, V. 2009b, 2011c,f,g).

Table 4: Binary moral operations "alethic faith" and "alethic doubt"

x	y	$F^{A}xy$	$D^{N}xy$	$F^{N}xy$	$D^{T}xy$	$S^{C}xy$	$N^{S}xy$
g	g	b	g	b	g	g	b
g	b	b	g	b	g	g	b
b	g	g	g	b	b	b	g
b	b	b	b	g	g	b	g

The formal-ethical relationship among these operations is represented below by the following formal-ethical hexagon containing the square of formal-ethical opposition as its aspect.

Picture 2: The Formal-Ethical Hexagon and the Square of Formal-Ethical Opposition 2



The glossary for the below table 5: Let symbol $N^{N}xy$ stand for the evaluation-function "x's *alethic* (true) *non-toleration of* (what, whom) *not-y*", or "x's *not-standing* (what, whom) *not-y*". $T^{O}xy$ stands for the evaluation-function "x's *alethictoleration of y*", or "x's *standing* (what, whom) *y*". $N^{O}xy$ stands for the function "x's *alethicnon-toleration of y*", or "x's *not-standing* (what, whom) *y*". $T^{N}xy$ – "x's *alethictoleration*

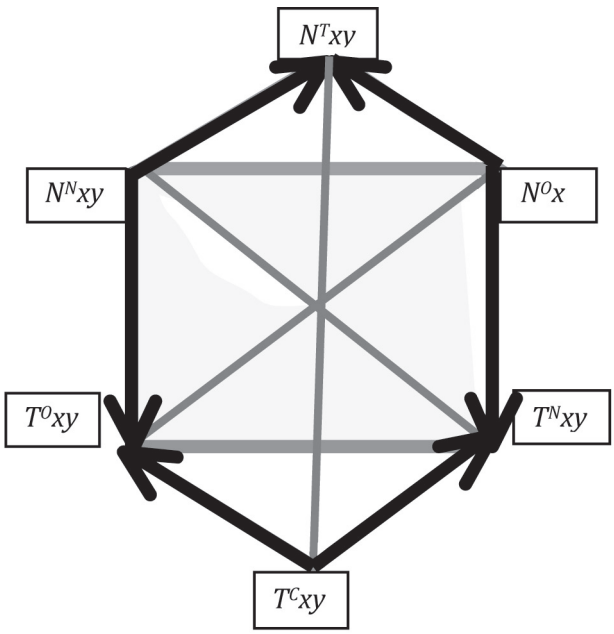
of not-y", or "x's *standing* (what, whom) *not-y*". $T^{C}xy$ – "x's *alethic tolerance concerning y, i.e.* x's *standing both: y and not-y". $N^{T}xy$ –*"nonbeing of x's *alethic tolerance concerning y", i.e.* "*either* x's *alethicnon-toleration of y*", or "x's *alethic non-toleration of not-y*". The operations are defined by the table 5.

Table 5: Differentbinary moral operations "toleration" and "tolerance"

x	y	$N^{N}xy$	$T^{O}xy$	$N^{O}xy$	$T^{N}xy$	$T^{C}xy$	$N^{T}xy$
g	g	b	g	b	g	g	b
g	b	b	g	b	g	g	b
b	g	g	g	b	b	b	g
b	b	b	b	g	g	b	g

The formal-ethical relationship among these moral operations is squared and hexagonized as follows.

Picture 3: The Formal-Ethical Hexagon and the Square of Formal-Ethical Opposition 3

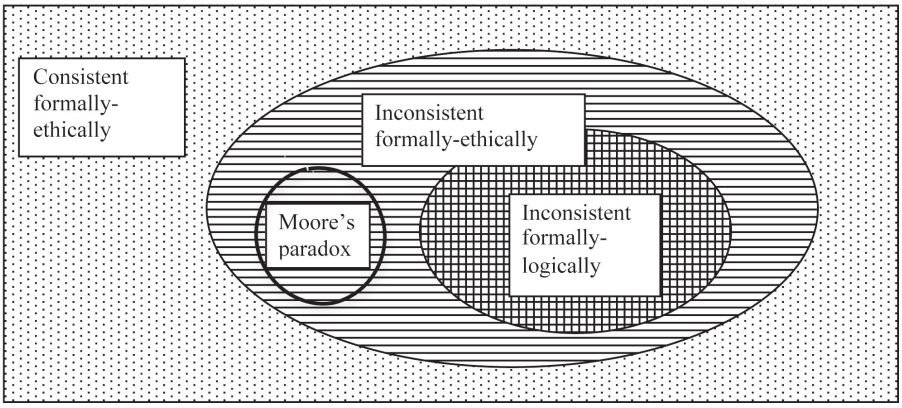


Systematically using the above-given definitions one can generate the following equations: 12) $K^{E}xy=+=F^{I}xy=+=N^{N}xy$; 13) $T^{C}xy=+=S^{C}xy=+=D^{X}xy$; 14)

Ty=+=T^C by=+=b; 15) *Gy=+=NTy=+=N^T by=+=g*; 16) *T^N xy=+=NFxy=+=D^T xy*; 17) *By=+=Ey;* 18) *NEy=+=Ny;* 19) *By=+=Fy;* 20) *NFy=+=Ny;*21) *Ey=+=NENy*;22) *Ey=+=NFNy*; 23) *Fy=+=NENy*;24) *Fy=+=NFNy*; 25) *Fy=+=NSNy=+=HNy*;26) *Ey=+=NSNy=+=HNy*; 27)*ENy=+=HFy*; 28) *FNy=+=HEy*; 29)*Fy=+=HENy*; 30) *Ty=+=Iy=+=b*. 31) *Dy=+=Gy=+=g*. 32)$K_2K^E xyNF^I xy=+=b$.

Generating these equations by means of computing relevant moral-evaluation-tables is not a purely scholastic exercise. Such generating formal-ethical equationscan have a heuristic significance for philosophy. For instance, the last equation means that the well-known epistemic paradox of George Edward Moore (See the article "Moore's paradox" in *Wikipedia*) is an example of the above-defined *formal-ethical contradiction* (of activity). See the definition *DF-4*. This result explains the fact that the strange sentence of Moore is considered as a paradox (formal contradiction) in spite of the fact that there is no formal-logical inconsistency in it. Notwithstanding the formal-logical consistency of Moore's exotic sentence (See Green, Mitchell S. and Williams, John N. 2007; Lobovikov, V. 2014a), that sentence does make up a paradox (formal contradiction), namely, the *formal-ethical contradiction* of activity. This statement of mine may be illustrated graphically by the following picture.

Picture 4: The graphic model of interrelations among the notions



Now let us depart from the realm of abstract ontology and epistemology to the evidently moral philosophy of proper love and hatred, fear and bravery. However the indicated shift of our attention is to be performed within the framework of algebra of formal ethics under review.It means that below we are to investigatelove, hatred, fear, etc. as *moral-action-forms*, i.e. *moral-evaluation-functionsdetermined by moral-evaluation-variables* in the proper mathematical meanings of the words "*function*" and "variable". To realize this intent with respect to love and hate it is necessary to introduce new symbols by means of the following glossary and precisely to define their moral-value-functional meanings by means of the below table 6.

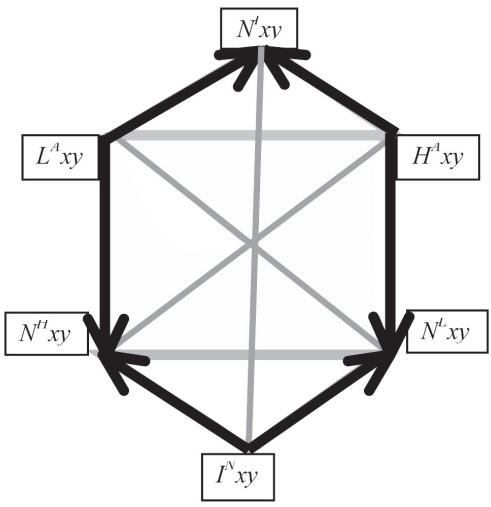
The glossary for the below-given table 6: Let symbol $L^{A}xy$ stand for moral-evaluation-function "x's *alethic* (true) *love* to/for (what, whom) *y*". The symbol $N^{H}xy$ stands for the function "nonbeing of x's (*alethic*) *hatred* to/for (what, whom) *y*". $H^{A}xy$ stands for "x's *alethic* (true) *hatred* to/for (what, whom) *y*". $N^{L}xy$ – "nonbeing of x's (*alethic*) *love* to/for (what, whom) *y*". $I^{N}xy$ – "x's *indifference* to (what, whom) *y*" or "x's *being moved neither byy nor by not-y*". $N^{I}xy$ – "x's *non-indifference* to (what, whom) *y*" or "x's *being moved either by y or by not-y*". The moral-evaluation-functional sense of these binary operations of two-valued algebra of formal ethics is precisely defined by the following *Table 6*.

Table 6: The functions "love" and "hatred"

x	y	$L^{A}xy$	$N^{H}xy$	$H^{A}xy$	$N^{L}xy$	$I^{N}xy$	$N^{I}xy$
g	g	b	g	b	g	g	b
g	b	b	g	b	g	g	b
b	g	g	g	b	b	b	g
b	b	b	b	g	g	b	g

This tabular definition of the binary moral operations "love", "hate" (and the other) in algebra of formal ethics was published in my monographs (Lobovikov, V. 2007b, 2009c) and papers (Lobovikov, V. 2011d,e). The formal-ethical relationship among themoral operations defined by the table 6 is represented below by the following formal-ethical hexagon containing the square of formal-ethical opposition as its aspect.

Picture 5: The Formal-Ethical Hexagon and the Square of Formal-Ethical Opposition 4



It is easy to see that (according to the *Table 6* and Picture 5) the functions $L^{A}xy$ and $H^{A}xy$ are *not contradictory but contrary* to each other. This is in accordance with the statement of Katarzyna Gan-Krzywoszynska and Piotr Lesniewski (2014, p. 66).

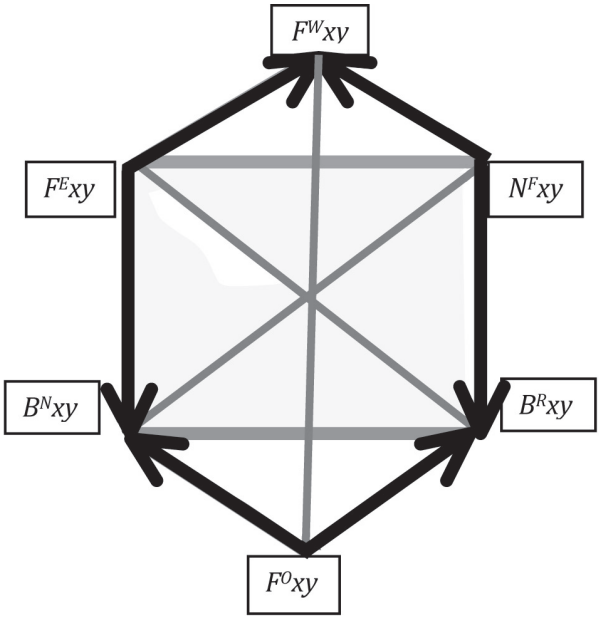
In many contexts (especially in psychological, moral and theological ones) love is compared not only with hate and emotional indifference but also with fear and courage. Some ordinary people tend to oppose love and fear, but this tendency is controversial. Therefore, here it is worth explicating the moral-value-functional meanings of the terms "*alethic* (true) *fear*" and "*being actually* (truly) *brave* (courageous)".

The glossary for the below-given table 7: Let symbol $F^{E}xy$ stand for moral-evaluation-function "x's *alethic* (true) *fear* of (what, whom) *y*". The symbol $B^{N}xy$ stands forthe function "nonbeing of x's (*alethic*) *fear*of (what, whom) *not-y*". $N^{F}xy$ stands for "x's *alethic* (true) *fear*of (what, whom) *not-y*". $B^{R}xy$ – "x's *alethic* (true) *bravery* in relation to *y*, i.e. nonbeing of x's (*alethic*) *fear*of (what, whom) *y*". $F^{O}xy$ – "x's *absolute fearlessness* concerning (what, whom) *y*" or "x's *fearing neither y nor not-y*". $F^{W}xy$ – "x's *not-absolute fearlessness* concerning (what, whom) *y*" or "x's *fearing either y or not-y*". The moral-evaluation-functional sense of these binary operations of two-valued algebra of formal ethics is precisely defined by the following *Table 7*.

Table 7: The functions "fear" and "bravery"

x	*y*	$F^{E}xy$	$B^{N}xy$	$N^{F}xy$	$B^{R}xy$	$F^{O}xy$	$N^{I}xy$
g	g	b	g	b	g	g	b
g	b	b	g	b	g	g	b
b	g	g	g	b	b	b	g
b	b	b	b	g	g	b	g

Picture 6: The Formal-Ethical Hexagon and the Square of Formal-Ethical Opposition 5



By means of the above-given definitions it is easy to demonstrate the following formal-ethical equations in two-valued algebra under investigation.

33) $L^{A}xy{=}{+}{=}F^{E}xy$. *34)* $K_{2}L^{A}xyNF^{E}xy{=}{+}{=}b$.*35)* $K_{2}F^{E}xyNL^{A}xy{=}{+}{=}b$.

They show that the above-mentioned tendency to oppose love and fear is not well-grounded. In particular, that controversial tendency contradicts to theology insisting that both (one's love for God) and (one's fear of God) are good (acts of the one).

According to the above-generated equations, the love-related moral categories are formally- ethically equivalent to the correspondingfear-related ones. Moreover, theabove-investigated alethic-knowledge-modalities and the corresponding true-faith-ones are formally-ethically equivalent to each other respectively. This outcome of discrete mathematical modeling is surprising for those philosophers who are used to the distinction (and even to the opposition) of "knowledge" and "belief", "love" and "fear", hence, these philosophers could protest against some of the above-listed equations and estimate them as paradoxes. However, in my opinion, there are only *illusions* of paradoxes caused by the ambiguity of the "natural" language of philosophers. For destroying such illusions of paradoxes in algebra of formal ethics there is a *formal principle of autonomy of facts and values,* which is precisely formulated as follows.

Let *ßx* stand for an act of informing (true or false affirming) that *x* takes place in reality. Concerning the relationship between "=+=" and "logic equivalence", the principle in question may be formulated as the following rule (A & B):

(A) From the truth of $x=+=y$ it does not follow logically that logic equivalence of βx and βy is true;

(B) From the truth of logic equivalence of βx and βy it does not follow logically that $x=+=y$ is true.

The *illusion* of "paradoxical" character of the above-listed formal-ethical equations is destroyed by the rule (A & B). This illusion is a result of not-recognized "jumps" from formal-ethical equivalences of moral evaluations to formal-logical equivalences of facts (and back from the formal-logical equivalences of facts to the formal-ethical equivalences of moral evaluations). In algebra of formal ethics such allegedly logical bridging the gap between facts and values is strictly forbidden by the rule (A & B). This rule is an explication of important particular case of *the general principle of mutual formal-logic autonomy of corresponding facts and values (propositions and evaluations)*. The rule (A & B) can be universalized in the following way. Let us call this generalization "the rule (Y & Z)";

(Y) From $x=+=y$ it does not logically follow that $(\beta x © \beta y)$;

(Z) From $(\beta x © \beta y)$ it does not logically follow that $x=+=y$.

In the statements (Y) and (Z), the symbol "©" stands for any element of the set of all *binary* formal logic operations.

As to the *unary* moral operations of algebra of formal ethics, the general value-&-fact-autonomy principle can be precisely formulated as the following rule U&Q:

U) From $\beta@x$ it does not logically follow that βx;

Q) From $@x$ it does not logically follow that $\beta@x$;

In the statements U) and Q), symbol "@" stands for any element of the set of all *unary* operations of algebra of formal ethics.

The general fact-&-value-autonomy principle U & Q may be exemplified by the so-called "Guillotine of D. Hume". Obviously, this naming is conventional: David Hume did not formulate the principle in such a way. He discussed an important *particular* case of the above-submitted *general* principle U & Q.

Let "U-GH" and "Q-GH" stand for those specific particular cases of U and Q respectively, which were discussed in the Guillotine of Hume. They may be formulated as follows:

U-GH: From βOx it does not logically follow that βx;

Q-GH: Fromβx it does not logically follow that βOx.

In the above-formulations, the symbol "*Ox*" stands for "*making x obligatory*" or "*x's being obligatory*".

From the viewpoint of history of philosophy, the names "U-GH" and "Q-GH" are also *conventional*: they are names of explications and symbolic representations of the *negative* answer to the question "Is it possible formally-logically to derive

corresponding obligations from facts (and corresponding facts – from obligations)?" Hume himself did not formulate the negative answer manifestly. The negative answer was ascribed to him by some of his proponents or opponents. Hume (1978) himself only recognized the very important problem and manifestly formulated it at the level of natural language. Nevertheless nowadays the expression "Guillotine of Hume" is associated with the negative answer to the above-mentioned question which (question) naturally arises in relation to that very small part of text of Hume's "Treatise of Human Nature", which is called "Hume's Guillotine".

BIBLIOGRAPHY

Béziau, J.-Y. & Payette, G. (eds.). 2012. *The square of opposition: a general framework for cognition*. Bern; N.Y.: Peter Lang.

Béziau, J.-Y. 2012a. "The New Rising of the Square of Opposition". *Around and Beyond the Square of Oppositon,* pp. 3-19. Béziau J.-Y. & Jacquette D. (eds.): Basel, 2012: Birkhäuser.

Béziau, J.-Y. 2012b. "The Power of the Hexagon". *Logica Universalis*. Vol. 6. No. 1-2. pp. 1-43.

Correia, M. 2009. "The Syllogistic Theory of Boethius". *Ancient Philosophy*. Vol. 29. Issue 2. pp. 391-415.

Correia, M. 2012. "Boethius on the Square of Opposition". *Around and Beyond the Square of Opposition*, pp. 41-52. Béziau J.-Y. & Jacquette D. (eds.). Basel, 2012: Birkhäuser.

Correia, M. 2014. "The Didactic and Theoretical Expositions of the Square of Opposition in Aristotelian Logic". *Handbook of the Fourth World Congress on the Square of Opposition,* p. 16. Vatican City 2014. J.-Y. Béziau & K. Gan-Krzywoszynska (eds.): Publishing House "Kontekst".

Desclés, J.-P. & Pascu, A. 2012. "The Cube Generalizing Aristotle's Square in Logic of Determination of Objects". *Around and Beyond the Square of Opposition,* pp. 277-291. Béziau J.-Y. & Jacquette D. (eds.). Basel, 2012: Birkhäuser.

Dufatanye, A.-A. 2012. "From the Logical Square to Blanché's Hexagon: Formalization, Applicability and the Idea of the Normative Structure of Thought". *Logica Universalis*. Vol. 6. No. 1-2., pp. 45-67.

Gan-Krzywoszynska, K. & Lesniewski, P. 2014. "Rationality and Indifference. On a Love-Hate Hexagon in the Foundations of Humanities". *Handbook of the Fourth World Congress on the Square of Opposition*, p. 66. Vatican City, 2014. J.-Y. Béziau and K. Gan-Krzywoszynska (eds.). Poznan-Poland: Publishing House "Kontekst".

Green, Mitchell S. and Williams, John N. 2007. (eds.) *Moore's Paradox: New Essays on Belief, Rationality and the First-Person.* Oxford: Clarendon Press. (New York: Oxford University Press).

Hume, D. 1978. *Treatise of Human Nature*, Oxford: Clarendon Press. (New York: Oxford University Press).

Lobovikov, V. 2007a. "Aristotelian and juridical modalities: a new theory of their unity. (A two-valued algebra of formal natural-law philosophy of modalities as moral-legal evaluation-functions determined by two variables – a complement to G.H. Wright's deontic logic interpretation of G.W. Leibniz's idea about the unity of the two kinds of modalities)." *Brazilian Legal Theory Review – RFDCL* (*Revista da Faculdade de Direito de Conselheiro Lafaiete* –Nova fase– Conselheiro Lafaiete. v.3. pp. 181-187.

Lobovikov, V. 2007b. *Mathematical Ethics, Metaphysics, and the Natural Law. (Algebra of metaphysics as algebra of formal axiology)*. Yekaterinburg, 2007: The Institute of Philosophy and Law of the Ural Division of the Academy of Sciences.

Lobovikov, V. 2009a. "Mathematical Logic as a Particular Case of Mathematical Ethics (Algebra of Formal Ethics as a Generalization of Algebra of Formal Logic)". *Proceedings of the 7th Panhellenic Logic Symposium (PLS7),* pp. 109-111. Greece, 2009. Costas Drossos; Pavlos Peppas; Constantine Tsinakis (eds.): Patras University Press.

Lobovikov, V. 2009b. "Modal Logic and Formal Axiology of Alethic and Epistemic Modalities". *Proceedings of the 7th Panhellenic Logic Symposium (PLS7),* pp. 112-116. Greece, 2009. Costas Drossos; Pavlos Peppas; Constantine Tsinakis (eds.): Patras University Press.

Lobovikov, V. 2009c. *Poverty of Nietzsche's philosophy and overcoming it by digital metaphysics. (A discrete mathematical simulation of Nietzsche's philosophy of consciousness, of religion, of morals, and of crime)*. Yekaterinburg, 2009: The Institute of Philosophy and Law of the Ural Division of the Academy of Sciences.

Lobovikov, V. 2009d. "Mathematical simulating formal axiological semantics of natural languages (A fundamental generalization of mathematical philosophy: from truth-values to axiological ones)". *Philosophy, mathematics, linguistics: aspects of interconnection. Proceedings of the International scientific conference,* pp. 128-132. (Sankt-Petersburg, 2009). L. Euler International mathematical institute (eds.): VVM.

Lobovikov, V. 2009e. "Algebra of formal logic as a particular case of algebra of formal ethics (Simulating moral-legal aspect of human conduct by operations of AI-robots)". *6th Smirnov readings in logic: Proceedings of the International scientific conference,* pp. 73-74 (Moscow, June, 2009). V.A. Bocharov, J.V. Ivlev, D.V. Zaytsev, A.S. Karpenko, O.M. Grigoryev (eds.); Chief-editor – V.I. Markin. M.: Contemporary copybooks.

Lobovikov, V. 2010a. "A Square of Opposition of Moral-Legal Evaluation-Functions in Two-Valued Algebra of Formal Ethics". *Handbook of the Second World Congress on Square of Opposition,* pp. 62-63. Corsica, June, 2010. J.-Y. Béziau & K. Gan-Krzywoszynska (eds.): Publishing House "Kontekst".

Lobovikov, V. 2010b. "Between writing and picturing: The square of opposition in formal logic and formal axiology of alethic and deontic modalities". *Image and Imaging in Philosophy, Science, and the Arts.* Papers of the 33rd International Wittgenstein Symposium, vol. XVIII, pp. 183-185 (Austria, 2010). Elisabeth Nemeth; Richard Heinrich; Wolfram Pichler (eds.): Austrian Ludwig Wittgenstein Society.

Lobovikov, V. 2010c. "Algebra of the Natural Law. A Mathematical Simulation of the Natural Law Type of Legal Reasoning: A Basis for Computer-Aided Law-Making and Law Education". *Archiv für Rechts- und Sozialphilosophie (ARSP) – the International Journal of the IVR – the International Association for Philosophy of Law and Social Philosophy*. Vol. 122, pp. 105-114. Imer B. Flores and Gülriz Uygur (eds.): Franz-Steiner-Verlag/Nomos.

Lobovikov, V. 2011a. "Discrete Mathematical Representing G.W. Leibniz' Philosophy of Law, Morals and Theology by Means of Two-valued Algebra of the Natural Law (Studying moral-legal evaluation-functions determined by two variables in algebra of formal axiology)". *XI Internationaler Leibniz-Kongress unter der Schirmherreschaft des Bundespräsidenten Natur und Subject.* Vorträge 2.Teil, pp. 611-615. Herbert Breger, Jürgen Herbst und Sven Erdner (eds.). Hannover, 2011. Berlin: Akademie Verlag; Bahlsen GmbH & Co. KG.

Lobovikov, V. 2011b. "The logical square in formal logic and an ethical square in formal ethics: their alethic, deontic and quantificational interpretations". *Logical-Philosophical Studies. Yearbook of the Association of Logicians of St.-Petersburg.* Issue 9. pp. 157-173. A.Y. Slinin and H.N. Lisanyuk (eds.). St.-Petersburg: The State University Press.

Lobovikov, V. 2011c. "Episteme and scientific knowledge as significantly different evaluation-functions in algebra of metaphysics (a law of contraposition of the two functions in metaphysics algebra)". *Proceedings Metaphysics 2009. 4th World Conference,* pp. 565-574, Rome, 2009. Paolo Zordan, Robert Badillo, Montserrat Lafuente (eds.). Madrid, 2011. Fondazione Idente di Studi e di Ricerca: Fundación José Ortega y Gasset

Lobovikov, V. 2011d. "Plato: formal-axiological aspects of the doctrine of love proper". *Herald of Novosibirsk State University. Series: Philosophy*, vol. 9, Issue 2. pp. 105-116.

Lobovikov, V. 2011e. "A formal-ethical law of contraposition of love: Plato's doctrine of two Aphrodites and two Eroses from the viewpoint of formal axiology". *Herald of Novosibirsk State University. Series: Philosophy*, vol. 9. Issue 4, pp. 38-48.

Lobovikov, V. 2011f. "Discrete Mathematical Representing the Value of Knowledge". *Epistemology: Contexts, Values, Disagreement. Proceedings of the 34th International Ludwig Wittgenstein Symposium in Kirchberg, 2011,* pp. 175-177. Christoph Jäger & Winfried Löffler (eds.). Kirchberg am Wechsel, 2011: Austrian Ludwig Wittgenstein Society.

Lobovikov, V. 2011g. "Science, episteme and mathematical ethics (A law of contraposition of episteme in algebra of formal ethics)". 14th International Congress of Logic, Methodology and Philosophy of Science. Volume of Abstracts, pp. 147-148. Nancy, France, 2011: Division of Logic, Methodology and Philosophy of Science of the International Union of History and Philosophy of Science.

Lobovikov, V. 2012. "Formal Ethics and Non-Monotonic Politics of Public Power and Private Property". *Papers of the 35th International Wittgenstein Symposium. Ethics – Society – Politics,* pp. 179-181, Kirchberg am Wechsel, Lower Austria, 2012). Martin G. Weiss & Hajo Greif (eds.): Austrian Ludwig Wittgenstein Society.

Lobovikov, V. 2013. "Private Vices Are Public Benefits: The Paradox of Bernard Mandeville as an Equation of Two-Valued Algebra of Formal Ethics". *Sins, Vices and Virtues: Essays*

in the Cultural, Political and Philosophical Reflections of Moral Categories, pp. 35-44. J. Midla & D. Schauffer (eds.) O x ford, Uninted Kingdom: Interdisciplinary Press.

Lobovikov, V. 2014a. "Moore's paradox in the light of algebra of formal ethics". *Tidings of the Ural Federal University. Series: Social Sciences*. N. 1. pp. 44-57.

Lobovikov, V. 2014b. "Squares and Hexagons for Moral-Evaluation-Functions 'Faith', 'Doubt', 'Knowledge', 'Assumption' and 'Toleration'in Algebra of Formal Ethics". *Handbook of the Fourth World Congress on the Square of Opposition,* pp. 98-99. J.-Y. Béziau & K. Gan-Krzywoszynska (eds.). Vatican City, 2014. Poznan-Poland: Publishing House "Kontekst". "Moore's paradox".

Wikipedia, the free encyclopedia. URL = <http://en.wikipedia.org/wiki/Moore%27s_paradox> (18.11.2013).

Prade, H. 2014. "A Hexagonal View of Artificial Intelligence". *Handbook of the Fourth World Congress on the Square of Opposition,* p. 22. Vatican City, 2014. J-Y. Béziau and K. Gan-Krzywoszynska. Poznan-Poland: Publishing House "Kontekst".

Woleński, J. 2014. "Lvov-Warsaw School". *The Stanford Encyclopedia of Philosophy* (Spring 2014 Edition), Edward N. Zalta (ed.). URL = <http://plato.stanford.edu/archives/spr2014/entries/lvov-warsaw/>. (June 14, 2014).