

# Probabilistic assessment of asphalt pavement design

## Determinación probabilística del tiempo de servicio de estructuras de pavimentos

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### Abstract

*In Chile, as in many South American countries, AASHTO-93 is the current official method for the design of asphalt pavements. The method gives an estimated number of 80 kN standard axle loads ( $W_{18}$ ) that the pavement can withstand for a certain serviceability. A safety factor is then applied to  $W_{18}$  to take into account the confidence level and the estimated variance of the inputs used in the method, whereby the magnitude of the structure's life cycle is estimated. Normally, data used in the AASHTO-93 method are assumed to be deterministic, disregarding their random nature. This paper presents a probabilistic assessment of the AASHTO-93 pavement design method under Chilean conditions, which provides a set of probability density functions (PDFs) that allow a probabilistic estimation of the pavement life cycle in terms of the value  $W_{18}$ . To achieve the goal, we developed a simulation model based on a sample of 13 groups of roads located throughout Chile.*

*Keywords: Pavement design, asphalt pavements, probabilistic assessment*

### Resumen

En Chile, como en muchos países de Suramérica, AASHTO 93 es el método oficial para el diseño de pavimentos asfálticos. El método entrega un número estimado de ejes equivalentes estándar de 80 kN que el pavimento puede soportar hasta alcanzar un cierto valor de serviciabilidad. Un factor de seguridad es luego aplicado al número estimado de ejes equivalente tomando en cuenta el nivel de confianza y la varianza estimada de los datos de entrada utilizados en el método, con lo cual se estima la magnitud del ciclo de vida de la estructura. Normalmente, los datos utilizados en el método AASHTO 93 son asumidos como valores deterministas, sin tomar en cuenta su naturaleza aleatoria. Este trabajo presenta una evaluación probabilística del método de diseño AASHTO 93 de pavimentos bajo condiciones chilenas, el cuál proporciona un conjunto de funciones de densidad de probabilidad, que permiten realizar la estimación probabilística del ciclo de vida del pavimento en términos del valor del número de ejes equivalente. Para alcanzar el objetivo se desarrollo un modelo de simulación a partir de una muestra de 13 grupos de carreteras ubicados a largo de Chile.

**Palabras clave:** Diseño de pavimentos, pavimentos en asfalto, evaluación probabilística

## 1. Introduction

*Pavement design methods are divided in three types based on their approach; they can be either mechanistic, when based on the mechanics of materials; empirical, when based on tests and field trials; or mechanistic empirical, when they adopt theoretical concepts from mechanics of materials complemented by field data (FHWA, 2006).*

*All design methods must incorporate reliability, so as to protect the design from the uncertainties that appear when defining traffic loads, environmental conditions, evolution of the deterioration, mechanical properties and structural performance of the materials (AASHTO, 1993), as well as the quality of the construction processes (Sánchez-Silva et al., 2005).*

*During the pavement's design stage, the reliability protecting the design from the uncertainty of the aspects mentioned above is defined as "the probability that a pavement section designed with this process will perform satisfactorily under traffic and environmental conditions during the period of life" (AASHTO, 1993).*

*Chile currently uses the AASHTO-93 design method (MOP, 2012), which assumes the design reliability by setting a confidence level ( $R$ ) that defines the level of variance adopted in the design ( $Z_R$ ), and a value that includes the variance estimated from each of the factors used in the model's prediction model (Huang, 1993).*

*The model's input and output data are defined in a deterministic way; however, it should be noted that pavement engineering projects have a certain degree of uncertainty in their design, construction and operation. This implies that an adequate performance of the solution cannot be absolutely guaranteed, only in terms of the probability of success to meet the performance criterion (Achintya and Sankaran, 2000), which in this case is the number of equivalent standard axle loads of 80 kN which define the pavement's service life.*

*The objective of this research is to incorporate probability when calculating the life cycle of a pavement structure. Therefore, a simulation model was developed to assess the probability of Chilean roads, classified by their geographic location, structural capacity and traffic, to reach the service life expressed in terms of the number of equivalent single axle loads of 80 kN.*

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## 2. Structural design methods

Pavements' structural design methods allow measuring the thickness of the pavement layers that withstand the action of vehicle loads and climate conditions during their life cycle, thereby guaranteeing the compliance with the serviceability thresholds previously defined. These design methods are divided in three types depending on their approach:

- The mechanistic methods assess the pavement's response through compressive stress, tensile stress and deformations (Tighe et al., 2007), relating structural behavior and performance in the presence of traffic loads and environmental conditions (FHWA 2006).
- The empirical methods are based on experimental results and they are represented by mathematical relations developed from field data (Carvalho and Schwartz, 2006). The AASHTO method is probably the most popular and most widely used empirical design method, not only in the USA, but also at global level (Abaza and Abu-Eisheh, 2003).
- The empirical mechanistic methods establish a relation between the pavement's response based on the materials mechanics, when they are subjected to traffic loads and climate conditions, and the pavement performance through empirical deterioration models (Carvalho et al., 2006), where the MEPDG method is one of the most representative.

Regardless of the structure design method selected, the variability of the model's input and output data must be considered together with the random factor in the construction processes, and the climate conditions to which the pavement structure is subjected to.

### 2.1 AASHTO-93 design method

The AASHTO-93 design method is based on algorithms developed in the AASHO road test, completed in May 1962 and modified in the design guides published on

July 1986 (Baus et al., 1989) and 1993. The design formula is represented by Equation 1.

$$\begin{aligned} \text{Log}(W_{18}) = & \\ Z_R * S_o + 9,36 * \text{log}(SN + 1) - 0,20 + & \frac{\text{log}\left(\frac{P_i - P_f}{4,2 - 1,5}\right)}{0,40 + \frac{1094}{(SN+1)^{5,19}}} + 2,32 * \\ \text{log}(M_R) - 8,07 & \end{aligned} \quad (1)$$

Where:

- $W_{18}$ : Predicted number of 80 kN equivalent single axle loads.
- $Z_R$ : Normal standard deviation associated with the design reliability R.
- $S_o$ : Combined standard error.
- $P_i$ : Initial serviceability index.
- $P_f$ : Final serviceability index.
- $SN$ : Structural number (inches).
- $M_R$ : Effective resilient modulus of the subgrade soil (MPa).

All factors in Equation 1 show some degree of uncertainty, but there are also other situations that are outside of the scope of the design technique, which also present variability. For example: the construction process, the climate condition of the site and the load applied to the structure. They all affect the adequate performance of the pavement throughout its life cycle.

The way the AASHTO-93 design methods takes into account the variability of these factors is based on the probability method (Lemer and Moavenzadeh, 1971). This method appreciates the reliability of the design by establishing a confidence level (R) that defines the level of variance adopted in the design ( $Z_R$ ) and a value including the variance estimated for each one of the factors used in the prediction model ( $S_o$ ) (Huang, 1993).

In the specific case of Chile, the values for R, Z and  $S_o$  are recommended by the Chilean Highway Design Manual (MOP, 2012), but in general they are defined based on the experience and good judgment of the designer, which can entail some risks of over or under sizing the structure, depending on the values adopted by the pavement designer (Figure 1a).

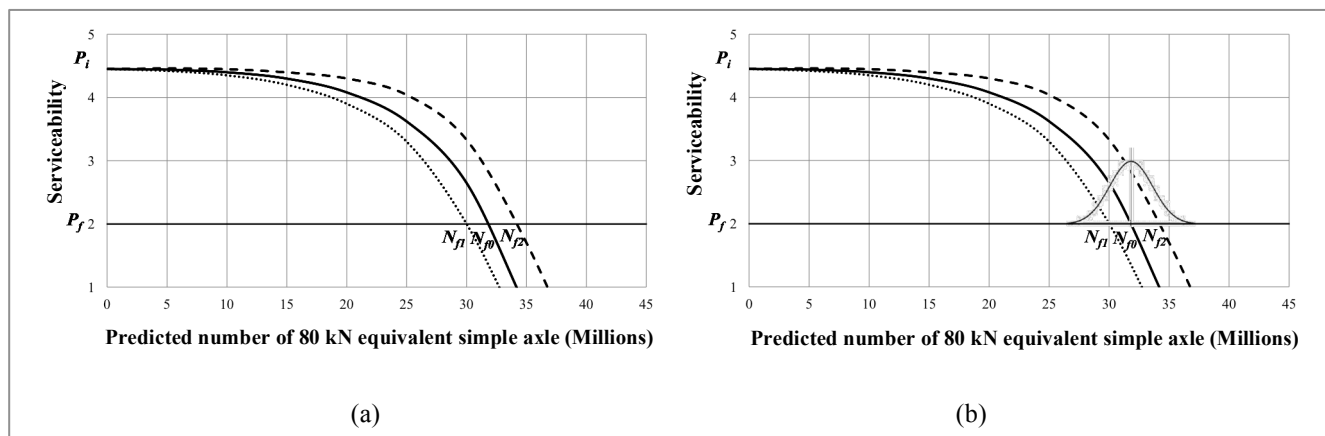


Figure 1. Serviceability of asphalt pavements versus pavement life ( $P_i$ =Initial serviceability,  $P_f$ =Final serviceability,  $N_{i1}$ =  $W_{18}$  for conservative designer,  $N_{i2}$ =  $W_{18}$  for less conservative designer,  $N_{i0}$ = real number of  $W_{18}$ )



The input data used in the design formula are defined according to the field data analysis, where the corridor is sectioned in order to delimit the design. In this regard, the input data of the design formula, and consequently the output data, are set in a deterministic way, which only applies a safety factor based on parameters  $R$ ,  $Z_R$  and  $S_w$ , to protect the design from the variability sources.

The reliability in the AASHTO-93 method is defined as "the probability that a pavement section designed with this process will perform satisfactorily under traffic and environmental conditions during the design period" (AASHTO, 1993). According to the latter, it seems more adequate to analyze the service life or life cycle under probabilistic hypotheses, as represented in Figure 1b, which is possible if the input data are considered as random variables, thereby obtaining an equally random output variable as a result, which represents the pavement's life cycle, expressed in terms of the number of 80 kN equivalent axle loads.

### 3. Methodology

With the aim of meeting the objective of this research, a simulation model was developed (Figure 2) using the Monte Carlo method, which could repeatedly evaluate the response of the AASHTO-93 design method for different road groups when their input variables were represented as probability density functions.

In order to define the input data of the design method, field data from roads located in Chile were used, which were in service for more than 10 years.

#### 3.1 Factorial design used in the simulation

The factorial design of the simulation considered three factors: geographic location, traffic and pavement structural capacity defined by the structural number. In turn, each factor was divided in three levels, which generated a factorial with 27 cells (Figure 3).

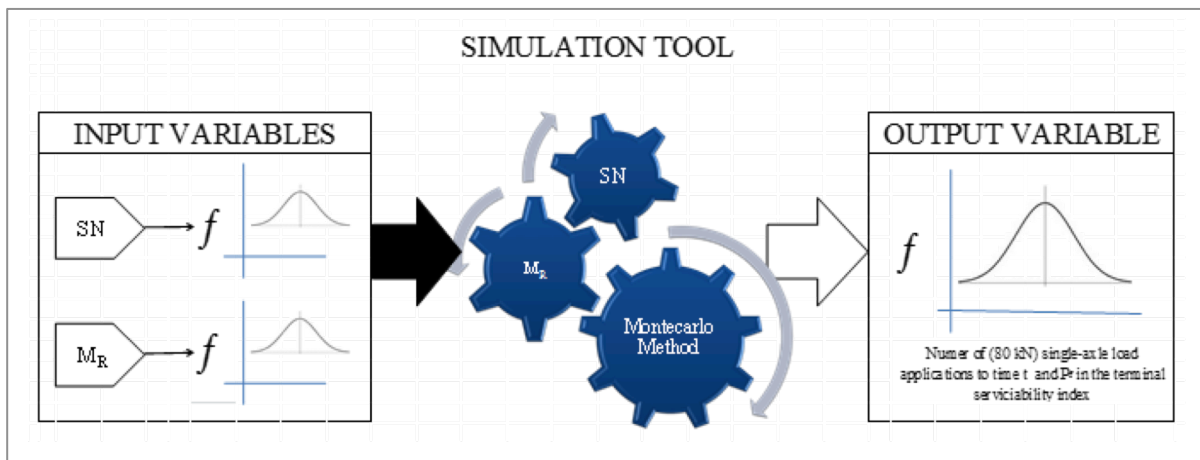


Figure 2. Execution of the simulation model

FACTOR ANALYSIS OF SIMULATION																												
Factors to Consider	Levels	Levels Classes																										
Geographic Location	3	North (N), Center (C), South (S)																										
Traffic	3	Low, Medium, High																										
Structure Type	1	Hot Mix Asphalt																										
Structural Number	3	Low, Medium, High																										
Hot Mix Asphalt	Geographic Location	North			Center			South																				
	Traffic	Low	Medium	High	Low	Medium	High	Low	Medium	High																		
	Structural Number	L	M	H	L	M	H	L	M	H	L	M	H	L	M	H	L	M	H									
Factorial cell number		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27

Figure 3. Factorial design matrix of the simulation model



The purpose was that each cell could be occupied with the largest number of asphalt pavement structures, thereby considering that highways had similar characteristics of geometry, structural capacity and traffic that would allow them to be grouped into a same factorial cell.

In the research, each road was divided in 20 meter long sampling units, which were selected at random and its final number was defined so that bias in the data could be avoided.

Each cell was identified by three groups of letters. The first group is composed by a character representing the geographic area to which the road belongs, north (N), center (C) or south (S). The second group represents the type of traffic, which can be low (LT), medium (MT) or high (HT).

The third group represents the value of the structural number as low (LS), medium (MS) or high (HS). For example, NLTLS represents a road located in the north, with low traffic and low structural number.

### 3.2 Simulation process

In order to prepare the simulation model, the first step was to establish the conceptual model under which the simulation was designed (Figure 4). The conceptual model was divided in three phases; phase 1 and 2 were developed simultaneously, while phase 3 gathered the results of the two previous phases.

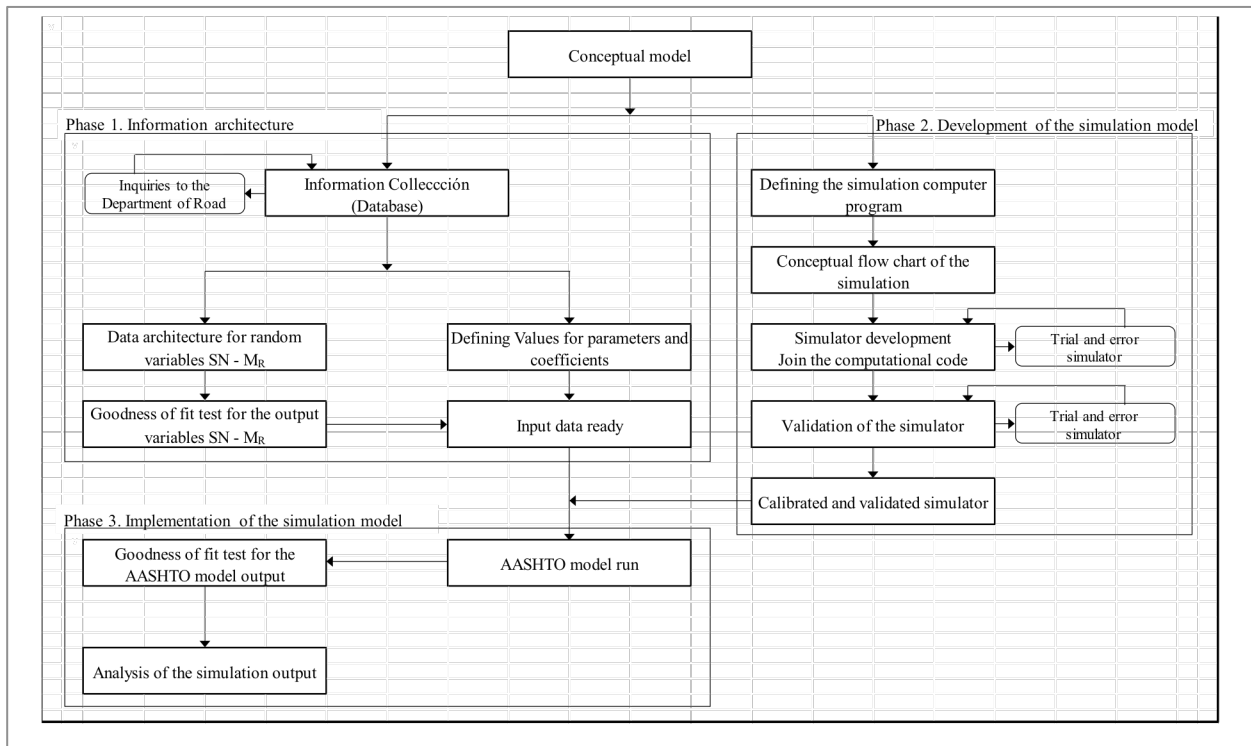


Figura 4. Conceptual model for the simulation model development

#### 3.2.1 Phase 1: data architecture (input data)

The input random variables were the Structural Number SN (in inches) and the subgrade's Resilient Modulus  $M_R$  (in MPa).

Detailed information was needed to calculate the structural number, which referred to the thickness and type of materials of each layer of the studied road sections, in addition to the subgrade's CBR value; these data were

obtained from the road network database of the Ministry of Public Works of Chile (MOP). Afterwards, the structural number was calculated based on Equation 2 (Huang, 1993) and Equation 3 (Hodges et al., 1975). The values of the structural and drainage coefficients were those recommended by MOP (2012).

$$SN = a_{11} * D_{11} + a_{12} * D_{12} + a_{13} * D_{13} + a_2 * D_2 * m_2 + a_3 * D_3 * m_3 + R \quad (2)$$

$$R = (3,51 * \log CBR) - [0,85 * (\log CBR)^2] - 1,43 \quad (3)$$

Where:



- **SN:** Structural number (in)
- $a_{ij}$ : Structural coefficient for the asphalt layer  $j$
- $D_{ij}$ : Thickness of the asphalt layer  $j$  (in)
- $a_2$ : Structural coefficient for the granular base layer
- $D_2$ : Thickness of the granular base layer (in)
- $m_2$ : Drainage coefficient of the granular base layer
- $a_3$ : Structural coefficient for the granular subbase layer
- $D_3$ : Thickness of the granular subbase layer (in)
- $m_3$ : Drainage coefficient of the granular subbase layer
- **R:** Contribution to the calculation of the subbase structural number
- **CBR:** Subgrade strength (California Bearing Ratio)

The probability density function represented by the random variable  $SN$  for each cell of the factorial design was obtained by finding out, through a statistical software, which was the probability density function that best fitted the structural number values calculated for all sections that were part of each factorial cell.

The Resilient Modulus of the subgrade was calculated with the CBR value of every pavement structure that was part of each factorial cell, based on the information from the MOP database. Equations 4 and 5 (MOP, 2012) were used in the calculation.

$$M_R = 17,6 * (CBR)^{0,64} \quad \text{For } CBR < 12\% \quad (4)$$

$$M_R = 22,1 * (CBR)^{0,55} \quad \text{For } 12 < CBR < 80\% \quad (5)$$

Where:

- $M_R$ : Resilient Modulus of the subgrade (MPa).
- **CBR:** Subgrade strength (California Bearing Ratio).

The probability density function represented by the random variable  $M_R$  for each factorial cell was obtained by finding out, through a statistical software, which was the probability density function that best fitted the  $M_R$  values calculated for all sections that were part of each factorial cell.

### 3.2.2 Phase 2: Development of the simulation model

In this phase, a software was selected to perform the simulation. Then, a flow chart with the moduli that took part in the simulation model was conceptually developed, and in the end the computer code was entered in each modulus using the software language.

The main structure of the simulation model was based on the Monte Carlo method, which allows assessing the risk or reliability of complex engineering systems by using random numbers associated to probability density functions, and that also allows determining the joint

probability when more than two random variables are incorporated to a problem, as in this case (Stewart, 2004).

The Monte Carlo method consists in choosing random numbers associated to a probability. This probability allows establishing the values of the variable needed to run the simulator in the density functions of the input variables. The output of the simulator represents a deterministic response of the phenomenon. This process is repeated a number of cycles that is set by selecting a confidence level for the response expected from the simulator. The set of outputs is later adjusted to a probability density function that best represents the model responses through a statistical test. This phase was completed with the calibration and validation of the simulation model.

### 3.2.3 Phase 3: Implementation of the Simulation Model

Once the simulation model was developed and input data were defined (random variables  $SN$  and  $M_R$ ), the simulator was launched using the input data established for each factorial design cell.

In order to set the value for the number of 80 kN equivalent axle loads that the structure could withstand according to the design formula, this stage of the simulation did not include the values for factors  $R$ ,  $Z_R$  and  $S_o$ ; instead, the basic expression of the AASHTO method was used.

The simulation was run a certain number of times calculated through the Student's t-test, with the aim of defining the size of a sample with 95% confidence. The set of data that make up the output of the different repetitions were adjusted to probability density functions and became the random variable that represents the probabilistic assessment of the prediction referring to the duration of the pavement's life cycle, expressed in terms of equivalent standard axle loads of 80 kN, according to the AASHTO-93 design method.

## 4. Analysis of Results

After analyzing the database provided by MOP, the data collected allowed the study of only 13 of the 27 cells from the initial factorial design, representing a database of 66 roads located along Chile (Figure 5).

### 4.1 Response of the Simulation Model

The response of the model provides a probability density function that predicts the service life in terms of the number of equivalent standard axle loads of 80 kN for each set of roads that was part of the factorial design matrix, which represent 13 probability density functions (Figure 6).

Figure 7 shows a sample of 6 from 13 responses (probability functions) concerning the AASHTO design method for different studied roads.



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Structural Number	3	Low, Medium, High																										
Hot Mix Asphalt	Geographic Location	North									Center									South								
	Traffic	Low			Medium			High			Low			Medium			High			Low			Medium			High		
	Structural Number	L	M	H	L	M	H	L	M	H	L	M	H	L	M	H	L	M	H	L	M	H	L	M	H	L	M	H
Factorial cell number		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27

Figure 5. Factorial design matrix of the simulation model actually used in the research

Factorial matrix cell	Probability density function of number of 80 kN equivalent single-axle load			
	Type Distribution	Parameters		
CHTLS	Lognormal	Mean = 69,7699	Test statistic	0,0364401
		Standard deviation = 94,8976	P-Value	0,14089
NHTMS	Gamma	Shape = 4,86458	Test statistic	0,0167036
		Scale = 0,0148237	P-Value	0,943211
CHTMS	Birnbaum Saunders	Shape = 0,778663	Test statistic	0,0355872
		Scale = 763,602	P-Value	0,159299
SHTMS	Birnbaum Saunders	Shape = 0,812225	Test statistic	0,0649037
		Scale = 56,3605	P-Value	0,00044235
NHTHS	Uniforme	límite inferior = 197,399	Test statistic	0,0686546
		límite superior = 354,651	P-Value	0,00016258
CHTHS	Birnbaum Saunders	Shape = 0,619581	Test statistic	0,0163219
		Scale = 834,228	P-Value	0,95895
SHTHS	Birnbaum Saunders	Shape = 1,15576	Test statistic	0,0479565
		Scale = 178,289	P-Value	0,0202035
CMTLS	Weibull	Shape = 0,929701	Test statistic	0,0288578
		Scale = 247,143	P-Value	0,38009
NMTMS	Weibull	Shape = 5,52287	Test statistic	0,0299494
		Scale = 273,967	P-Value	0,333976
SMTMS	Birnbaum Saunders	Shape = 0,845837	Test statistic	0,0471903
		Scale = 96,5518	P-Value	0,0233712
NMTHS	Weibull	Shape = 0,634069	Test statistic	0,015737
		Scale = 4210,74	P-Value	0,965585
SMTHS	Birnbaum Saunders	Shape = 0,245865	Test statistic	0,0350018
		Scale = 520,727	P-Value	0,173023
CLTMS	Birnbaum Saunders	Shape = 0,364578	Test statistic	0,0188991
		Scale = 111,545	P-Value	0,867783

Figure 6. Probabilistic response of the AASHTO design method



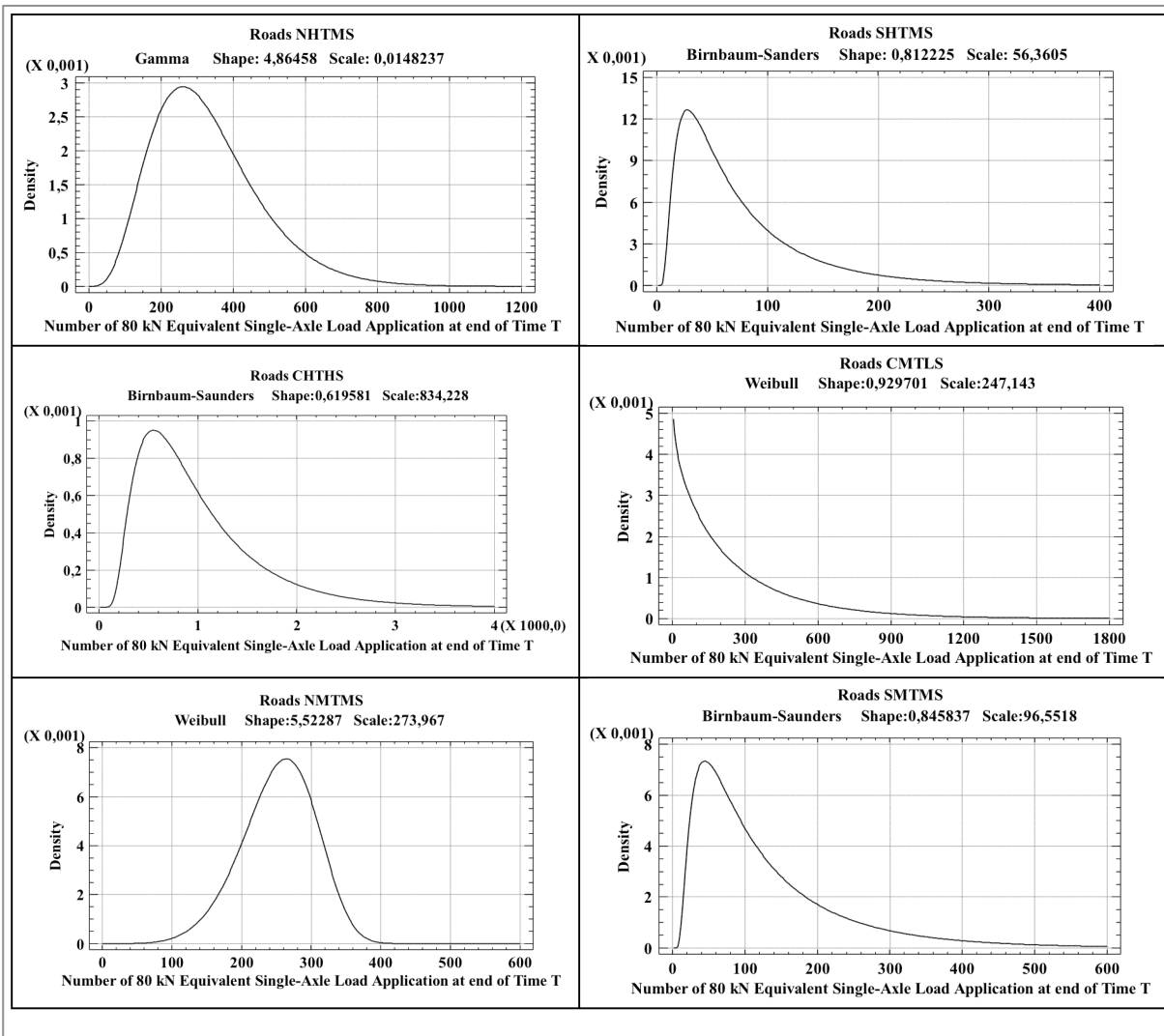


Figure 7. Probability density functions of the AASHTO design method

Figure 8 shows the average deterministic value of the number of equivalent single axle loads of 80 kN for each factorial cell. For calculation purposes, a 25-year traffic projection was made, with the help of the motor vehicle census data contained in the MOP database. Finally, the

equivalent axle load values were calculated for each studied road. The values regarding the traffic growth rate and the equivalent axle factors were used in previous local studies (De Solminihac et al., 2003; MOP, 2010) and they were defined by MOP based on historical records (Videla et al., 1996).

Factorial matrix cell	CHTLS	NHTMS	CHTMS	SHTMS	NHTHS	CHTHS	SHTHS	CMTLS	NMTMS	SMTMS	NMTHS	SMTHS
Numer of (80 kN) single-axle load applications to time t	30,4	19,1	16,6	18,9	12,3	23,4	20,2	6,1	7,0	5,4	10,3	6,7

Figure 8. Deterministic values of  $W_{18}$  for each group of roads



## 4.2 Discussion

In general, density plots present kurtosis and skewness and their analysis showed the following:

All responses of the model are asymmetrical, twelve present a positive skewness and one has a negative skewness, thereby indicating that the model has a response concentration skewed to the left or the right respectively. Most of the analyzed road groups present a data concentration to the left of the mode, which indicates that there is a higher probability for the model response to be below this value.

Eleven road groups showed positive kurtosis, indicating that the distribution is more peaked at the mode and has longer tails than usual and, therefore, it shows a high output data concentration level around the variable mode. Only one group presented a negative kurtosis, indicating that the function is flatter than the normal distribution with shorter tails; therefore, it shows a low output data concentration level around the variable mode.

The probabilistic response of the AASHTO design method for seven factorial groups are represented by probability functions of the Birnbaum-Sanders type, three by Weibull and the remaining three by Lognormal, Gamma and uniform distribution (Figure 6). All probability distributions, except the uniform one, correspond to life distribution functions that are a probability model associated to a life variable (Marshall and Olkin, 2007), which are characterized for being asymmetrical and positively skewed; they assess and predict the service life properly.

The Birnbaum-Saunders distribution is known as the fatigue life distribution, since it describes the materials' fatigue process from the initial stages of imperceptible cracks, followed by growth and propagation caused by cyclic stress and tension, to the materials' rupture or failure due to fatigue (Barros, Paula, Leiva, 2009), which adequately reflects the deterioration

suffered throughout the life cycle of an asphalt pavement structure.

Finally, when comparing the probabilistic results obtained in this research with the deterministic values of the number of equivalent axle loads that define the length of the structures' life cycle, it is possible to establish that, except for the CHTLS roads that have a 62% probability of failure before reaching the number of equivalent axle loads that define the life cycle, most of the structures will easily comply with the prediction, thereby showing that most structures are oversized.

## 5. Conclusions

- This paper developed a probabilistic model of the AASHTO-93 design method, using a Chilean database. The research provides expressions that predict the length of a pavement life cycle by using probability density functions in highways classified by geographic location, type of traffic and structural capacity.
- The methodology implemented in this study could be replicated in any part of the world, but the specific results only apply to Chilean road conditions, as a consequence of the database used in the research.
- The result of this research can be used to incorporate risk analysis in the design of pavement layers.

The functions describing the prediction of the pavement structures' life cycle in terms of the number of equivalent axle loads of 80 kN correspond mostly to life distributions, where the Birnbaum-Saunders type of functions are more frequent and they adequately describe the fatigue failure caused by physical phenomena.

## 6. References

- Achintya H. y Sankaran M. (2000)**, Probability, Reliability, and Statistical Methods in Engineering Design. New York: John Wiley & Sons, Inc. Cap. 5, 6, 7, 8.
- American Association of State Highway and Transportation Officials (1993)**, AASHTO Guide for Design of Pavement Structures. United States: (s.n.). Parte I: Cap. 1, 4. Parte II: Cap. 1, 2, 3.
- Abaza K. y Abu-Eisheh S. (2003)**, An Optimum Design Approach for Flexible Pavement. The International Journal of Pavement Engineering, 4 (1), 1-11.
- Barros M., Paula G.A. y Leiva V. (2009)**, An R implementation for generalized Birnbaum-Saunders distributions. Computational Statistics and Data Analysis, 53 (5), 1511-1528.
- Baus R. y Fogg J. (1989)**, AASHTO Flexible Pavement Design Equation Study. Journal of Transportation Engineering, 115 (5), 559-564.
- Carvalho R. y Schwartz C. (2006)**, Comparisons of Flexible Pavement Design AASHTO Empirical Versus NCHRP Project 1-37A Mechanistic-Empirical. Journal of Transportation Research Record, (1947), 167 - 174.
- De Solminihaç H., Hidalgo P. y Salgado M. (2003)**, Calibration of Performance Models for Surface Treatment to Chilean Conditions: the HDM-4 Case. Journal of the Transportation Research Board, (1819), 285-293.
- Federal Highway Administration FHWA (2006)**, Geotechnical Aspects of Pavements. Publication No. FHWA NHI-05-037. Washington: Federal Highway Administration. U.S. Department of Transportation. Cap. 1, 3, 5, 6.
- Hodges J. W., J. Rolt y T.E. Jones (1975)**, The Kenya Road Transport Cost Study: Research on Road Deterioration, TRRL Laboratory Report 673. Crowthorne, Inglaterra: Transport Research Laboratory. Cap. 1, 2, 3, 4, 5.
- Huang Y. (1993)**, Pavement Analysis and Design. (2a ed.). New Jersey: Prentice Hall. Capítulos 10 y 11
- Lemer A. C. y Moavenzadeh F. (1971)**, Reliability of highway pavements. Highway Research Record, (362), 1-8.
- Marshall A. y Olkin I. (2007)**, Life Distributions. Structure of Nonparametric, Semiparametric, and Parametric Families. Nueva York: Springer. Cap. 1, 2.
- Ministerio de Obras Públicas – MOP (2012)**, Manual de Carreteras. Santiago, Chile: Ministerio de Obras Públicas – Dirección de Vialidad. Cap. 3
- Ministerio de Obras Públicas de Chile (2010)**, Estudio Básico Seguimiento de Pavimentos Asfálticos. Santiago, Chile: Ministerio de Obras Públicas.
- Sanchez-Silva M., Arroyo O., Junca M., Caro S. y Caicedo B. (2005)**, Reliability based design optimization of asphalt pavements. International Journal of Pavement Engineering, 6 (4), 281- 294.
- Stewart Robinson (2004)**, Simulation: The Practice of Model Development and Use. Inglaterra: John Wiley and Sons, Inc. Cap. 1, 2, 5, 6, 7.
- Tighe S., Huen K. y Haas R. (2007)**, Environmental and traffic deterioration with mechanistic empirical pavement design model. Journal of Transportation Research Record, 2 (1989), 336-343.
- Videla C., De Solminihaç H., Gaete R. y Bustos M. (1996)**, Ajuste de Factores de Calibración para Ampliar Modelos de Deterioro de Pavimentos asfálticos. Santiago, Chile: Ministerio de Obras Públicas y Pontificia Universidad Católica de Chile.

